Dynamic Modeling of Selective and Nonselective Advertising Strategies

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Overview

- **Objective:** to model a competitive advertising situation where a firm gains customers from advertising effort and loses customers to competitors' advertising efforts and sales decay, leading to a method to determine the best allocation of advertising expenditures if competitors' customers are identifiable
- We construct two differential equation models of advertising competition:
  - **Nonselective:** Firms advertise equally towards all current non-customers.
  - **Selective:** Firms can allocate different expenditures towards customers of other firms and uncommitted customers.
- **Example application:** Is it better for a political candidate to advertise more heavily towards supporters of an opponent or undecided voters?

Introduction

- Firms are gaining the capability to discern competitors' customers from the market share
- Firms must determine the best way to allocate advertising expenditures
- Oligopoly setting: small number of firms $k = 1, \dots, n$
- Basic idea: $\begin{pmatrix}\text{change in}\\ \text{market share}\end{pmatrix} = \begin{pmatrix}\text{advertising}\\ \text{effectiveness}\end{pmatrix} \times \begin{pmatrix}\text{proportion of}\\ \text{market share}\end{pmatrix}$

Variables

$s_k(t)$	Sales of firm k at time t
$\varepsilon(t)$	Market potential at time t
$w_k(t)$	Effort towards all non-customers
τ_k	Effectiveness/effort ratio towards non-customers
$u_k(t)$	Effort towards competitors' customers
ρ_k	Effectiveness/effort ratio towards competitors' customers
$v_k(t)$	Effort towards market potential
σ_k	Effectiveness/effort ratio towards market potential
c_k	Sales decay rate

Nonselective Model

$$\underbrace{\dot{s}_k(t)}_{\text{Rate of change of market share}} = \underbrace{\tau_k w_k(t)\,(1 - s_k(t))}_{\text{Sales gain from advertising}} - \underbrace{s_k(t) c_k(t)}_{\text{Loss due to sales decay}} - \underbrace{s_k \sum_{j \neq k}^{n} \rho_j u_j(t)}_{\text{Loss due to competitors' advertising}}$$

Selective Model

$$\underbrace{\dot{s}_k(t)}_{\text{Rate of change of market share}} = \underbrace{\sigma_k v_k(t)\varepsilon(t)}_{\text{Sales gain from market potential}} + \underbrace{\rho_k u_k(t) \sum_{j \neq k}^{n} s_j(t)}_{\text{Sales gain from competitors' customers}} - \underbrace{s_k(t) c_k(t)}_{\text{Loss due to sales decay}} - \underbrace{s_k \sum_{j \neq k}^{n} \rho_j u_j(t)}_{\text{Loss due to competitors' advertising}}$$

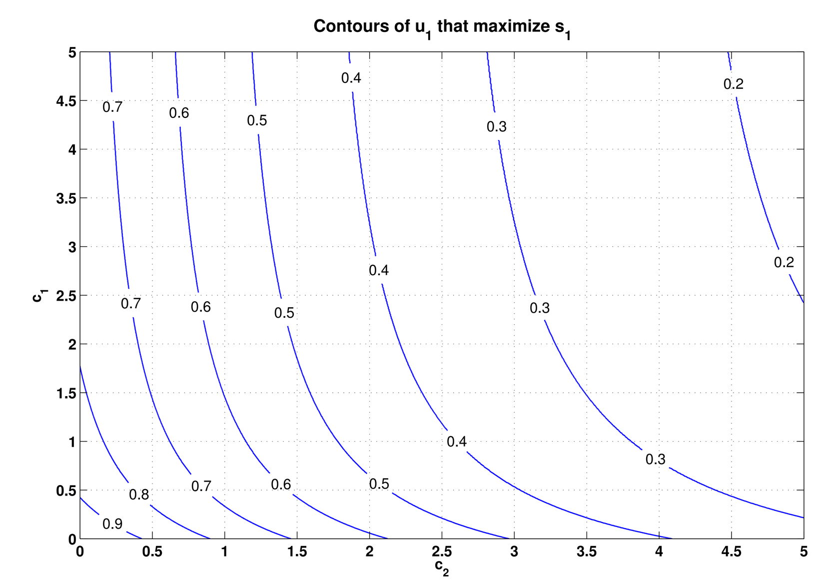
Selective vs. Nonselective

Given a single competitor with a nonselective advertising policy, which allocation of effort will **maximize** steady state market share?

Maximizing Market Share in a Duopoly

Theorem: When effectiveness/effort ratios and budgets are the same for both firms, the maximum market share for firm 1 is

$$s_1 = \frac{\frac{1}{\sqrt{2}}u_1 + (c_2 + u_1)\sqrt{1 - u_1^2}}{\left(\frac{1}{\sqrt{2}} + c_2 + u_1\right)\left(\frac{1}{\sqrt{2}} + c_1 + \sqrt{1 - u_1^2}\right)}.$$



Example: If $c_1 = c_2 = 1$, then $u_1 \approx 0.48$ maximizes s_1 and

$$s_1 \approx 0.2942, \qquad s_2 \approx 0.3030$$

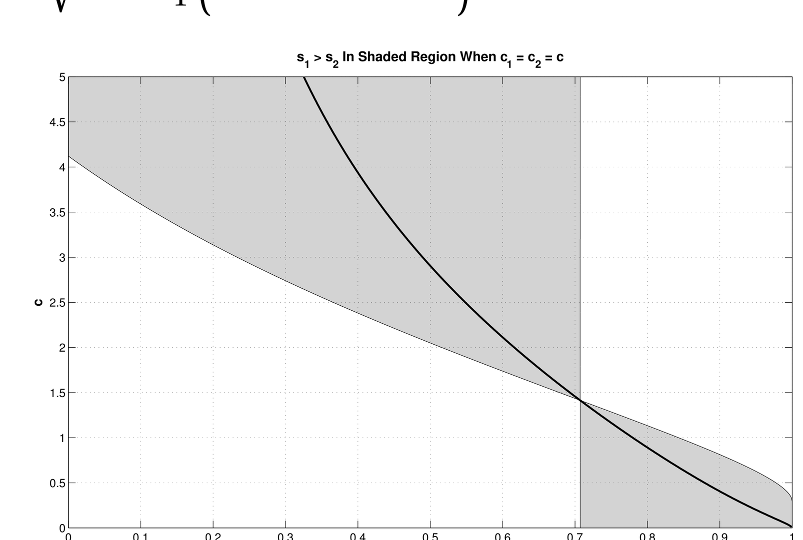
The selective strategy that maximizes sales rate/market share does not necessarily lead to higher sales than a competitor practicing nonselective advertising.

Under what conditions will a selective advertising policy result in an equilibrium market share **larger** than a nonselective competitor?

Beating a Nonselective Competitor

Theorem: When effectiveness/effort ratios and cancellation rates are the same for both firms, firm 1 (selective) will have a larger market than firm 2 (nonselective) whenever

$$\sqrt{1 - u_1^2}\left(2c + 2u_1 + \sqrt{2}\right) + \sqrt{2}(u_1 - c) > 1.$$



Example: If $c_1 = c_2 = 1$, then $u_1 = 0.78$ leads to steady market shares of

$$s_1 \approx 0.2870, \qquad s_2 \approx 0.2843$$

The curve within the shaded area indicates the value of u_1 that maximizes $s_1 - s_2$

Multi-Tier Product Extension

Scenario: A single firm offers a product/service at n levels or tiers. Let $k = 1, 2, \dots, n$ represent the n tiers of product/service, with $k = 1$ representing the highest tier and $k = n$ the entry-level tier.

Assumptions:

- The customers of each tier are mutually exclusive
- Profit margins are larger for higher tiers
- Uniform advertising effort $v(t)$ towards noncustomers
- The firm wishes to prevent downgrades in level, so higher tiers consider customers of lower tiers as potential customers, but not vice-versa.

The dynamics of the market share s_k of tier k are:

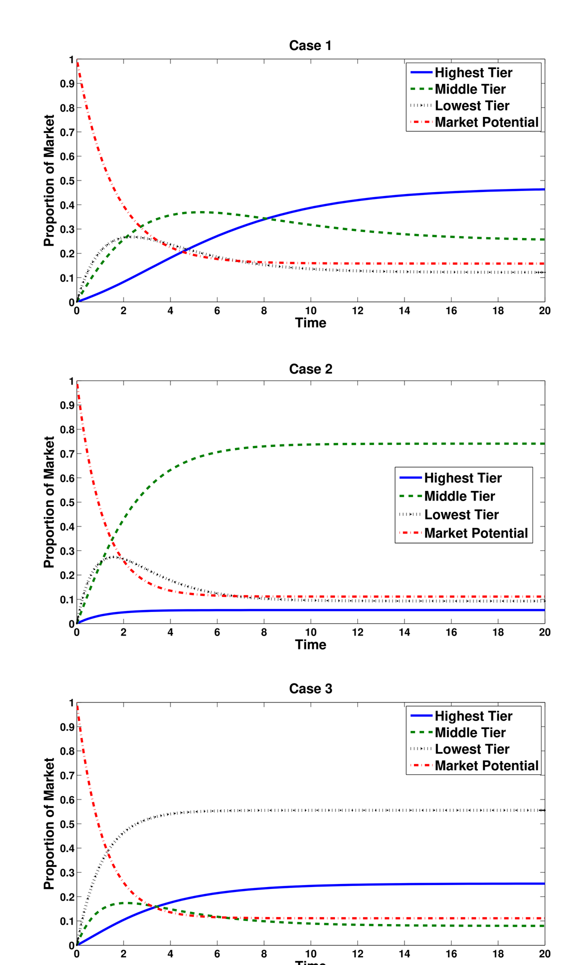
$$\dot{s}_k(t) = \underbrace{\sigma_k v(t)\varepsilon(t)}_{\text{Gain from market potential}} + \underbrace{\rho_k u_k(t) \sum_{j>k}^{n} s_j(t)}_{\text{Gain from upgrades from lower tier}} - \underbrace{s_k(t) c_k(t)}_{\text{Loss due to sales decay}} - \underbrace{s_k(t) \sum_{j<k}^{n} \rho_j u_j(t)}_{\text{Loss from upgrades to higher tiers}}$$

Three Tiers with Variable Effectiveness

Scenario: A firm offers three tiers of service and the primary advertising mechanism is accomplished through targeted mailings. Existing customers who upgrade their service only do so one level at a time.

Assumptions:

- New customers are more likely to sign up for the lowest-tier service than the middle tier, with the highest tier receiving the fewest new sign-ups.
- Mailings convincing lowest-tier customers to upgrade are generally more effective than those directed towards middle-tier customers
- All three tiers have the same cancellation rate



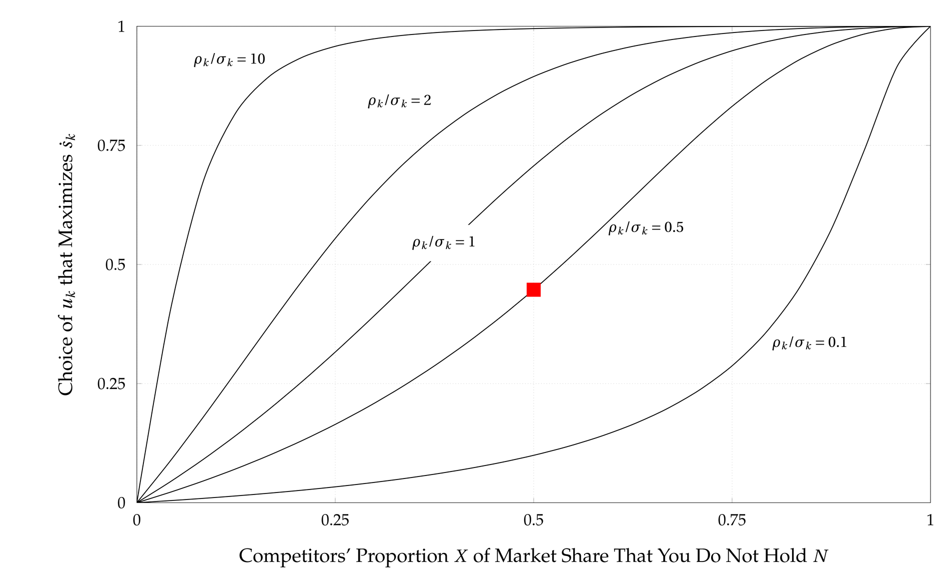
	Case 1	Case 2	Case 3
u_1	$1/\sqrt{3}$	0	$1/\sqrt{2}$
u_2	$1/\sqrt{3}$	$1/\sqrt{2}$	0
v	$1/\sqrt{3}$	$1/\sqrt{2}$	$1/\sqrt{2}$
s_1	0.464	0.056	0.254
s_2	0.257	0.741	0.080
s_3	0.122	0.093	0.556
ε	0.158	0.111	0.111

The model can be tuned to align with the firm's objectives. For example, a firm strictly concerned with market penetration would benefit from the advertising effort allotments in cases two and three.

Maximizing Rate of Increase of Sales/Market Share

Theorem: Let N represent the proportion of the market you do not hold and X represent the proportion of N that competitors hold. To maximize your current rate of increase of market share, choose the advertising effort u_k satisfying

$$u_k = \frac{\rho_k X}{\sqrt{\sigma_k^2(1 - X)^2 + \rho_k^2 X^2}}.$$



Example: if your competitors currently hold a 50% share of your non-customers, and your advertising towards the market potential is twice as effective as your advertising towards competitors' customers ($\rho_k/\sigma_k = 0.5$), then you should allocate 44.7% of your advertising budget towards competitors' customers and 55.3% towards uncommitted customers.

Conclusions and Future Work

- Extended the work in [1] and [2] to allow for differing allocation of effort towards market potential and competitors' customers.
- Optimal choices of allocation of advertising expenditures depend on sales decay rates and current market share.
- Maximizing your market share may not lead to a greater market share than competitors and vice versa.
- Future work: allow different allocations for multiple competitors, incorporate advertising cost and profit components for tiered products.

References

[1] Fruchter, Gila E., *Oligopoly advertising strategies with market expansion*, Optimal Control Applications and Methods, 20 (1999), pp. 199–211.

[2] Qinan Wang and Zhang Wu, *A duopolistic model of dynamic competitive advertising*, European Journal of Operational Research, 128 (2001), pp. 213 – 226.

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