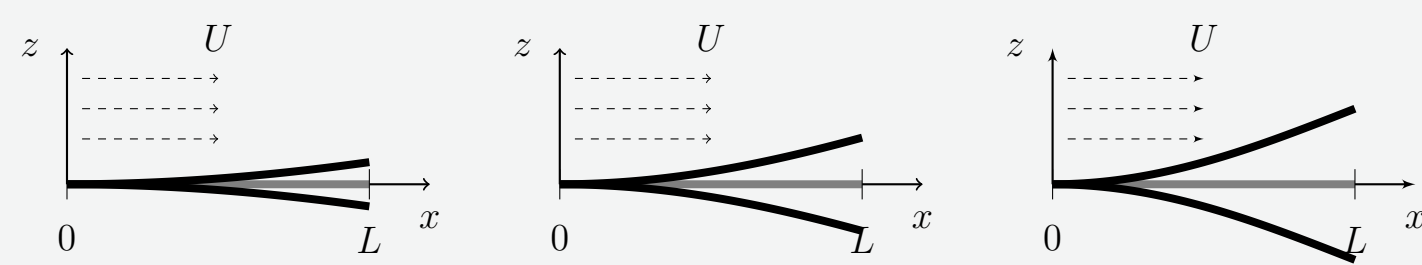
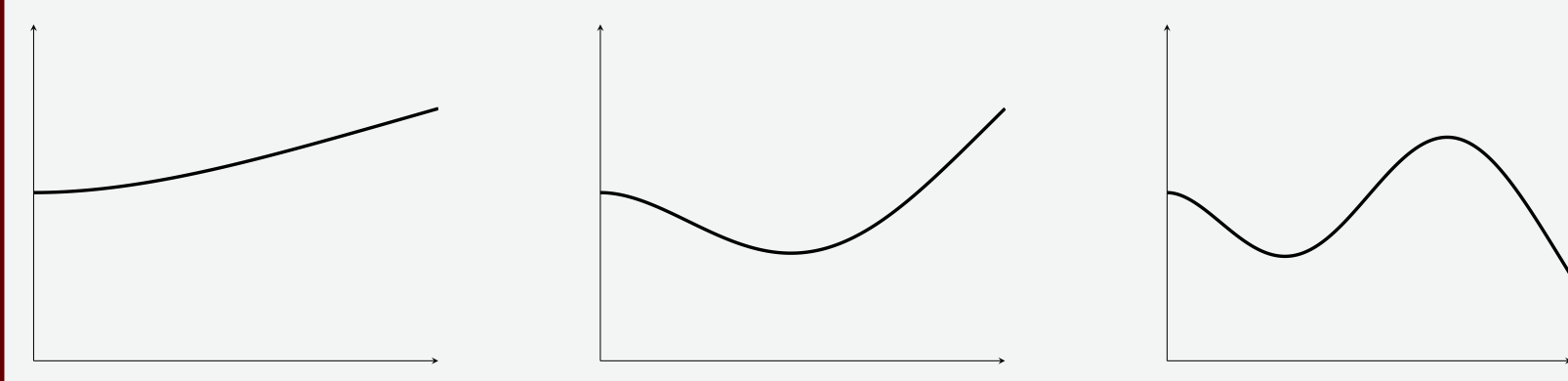


Introduction

- Flutter is a dynamic instability of an elastic structure in a fluid flow, caused by positive feedback between the body's deflection and the force exerted by the fluid flow.



- The elastic body has an infinite number of natural ways in which it wants to deflect due to external stresses.



Goals

- Understand parameter relationships
- Predict the onset of flutter using various computational methods
- Assess the validity of the piston-theoretic model.

Model

- Standard linear beam model with piston theoretic RHS.
- Piston Theory: assume that pressure on surface of plate can be treated as a piston acting on a narrow column of air above the plate.

$$\begin{cases} u_{tt} + D\partial_x^4 u + k_0 u_t = -\beta(u_t + U u_x) \\ u(t=0) = u_0; \quad u_t(t=0) = u_1 \\ BC(0); \quad BC(L). \end{cases} \quad (1)$$

- Piston Theoretic RHS

$$\frac{-\mu U}{\sqrt{U^2 - 1}}(u_t + U u_x)$$

- Parameter Log

- D : Stiffness parameter
- k_0 : Damping
- U : Fluid Flow Velocity
- L : Length of Beam

Clamped-Clamped BC:

$$u(0) = u_x(0) = 0, \quad u(L) = u_x(L) = 0$$



Hinged-Hinged BC:

$$u(0) = u_{xx}(0) = 0, \quad u(L) = u_{xx}(L) = 0$$

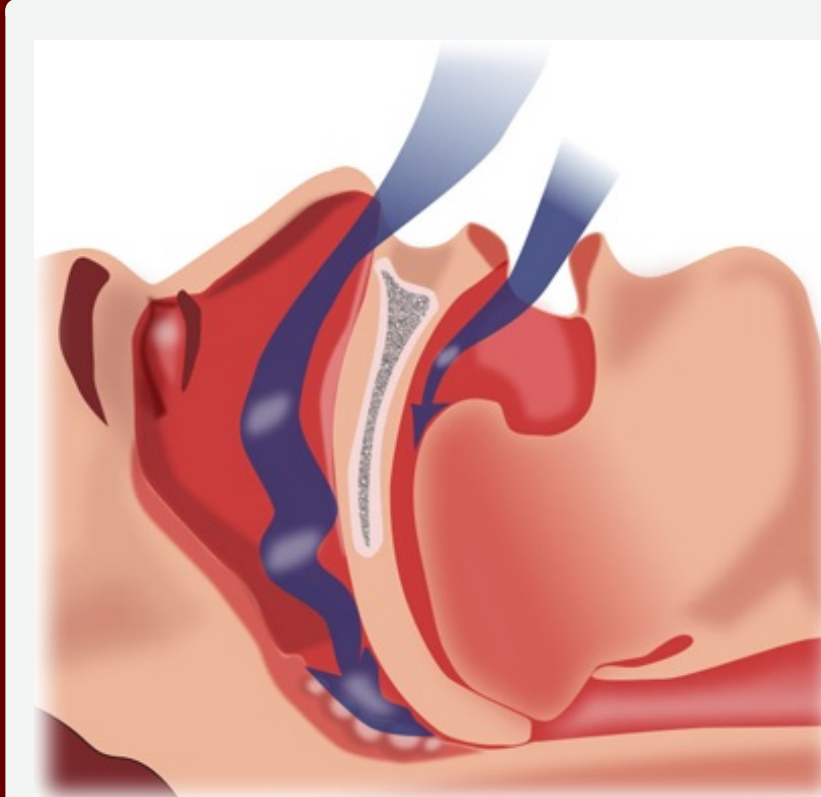


Clamped-Free BC:

$$u(0) = u_x(0) = 0, \quad u_{xx}(L) = u_{xxx}(L) = 0$$

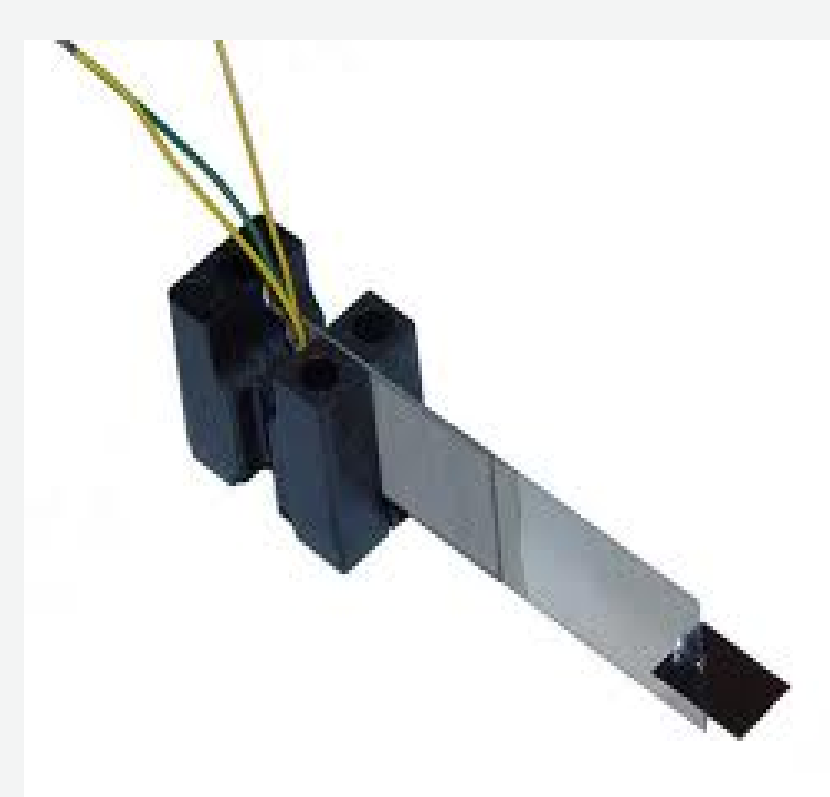


Motivation and Applications



- Aerospace: flaps and flags
- Biology: snoring and sleep apnea
- Piezoelectric Energy Harvesting

Clamped-free configuration not well understood. Peculiarities include: low flutter velocities and large displacements in the unstable regime.



Spatial and Temporal Discretization

- Approximate spatial derivatives using second-order Taylor series expansions:

$$\partial_x u \approx \frac{u(x+h) - u(x-h)}{2h}, \quad \partial_x^2 u \approx \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}$$

$$\partial_x^4 u \approx \frac{u(x+2h) - 4u(x+h) + 6u(x) - 4u(x-h) + u(x-2h)}{h^4}$$

- Boundary conditions implemented using ghost values outside domain.
- Model is reduced to first-order system in time:

$$\begin{bmatrix} u \\ v \end{bmatrix}' = \begin{bmatrix} v \\ -D\partial_x^4 u - k_0 v - \beta(v + U u_x) \end{bmatrix}$$

- Time integration: makes use of `scipy.integrate.ode` python package, employing backward differentiation formulas (BDFs).

Modal Analysis

- Separation of variables for $u_{tt} + D u_{xxxx} = 0$ in vacuo: assume $u(x, t) = S(x)f(t)$

$$S_{xxxx} = k_m^4 S, \quad f_{tt} = -\omega_m^2 f$$

- Use Euler identity $e^{ix} = \cos x + i \sin x$ to derive our basis functions $\cos, \sin, \cosh,$ and \sinh .

- Linear combinations of trig and hyperbolic trig functions comprise the basis functions/mode shapes:

$$S_n(x) = \cos(k_n x) - \cosh(k_n x) - d_n [\sin(k_n x) - \sinh(k_n x)]$$

- Solve eigenvalue problem to find shifted frequencies (modes): assume

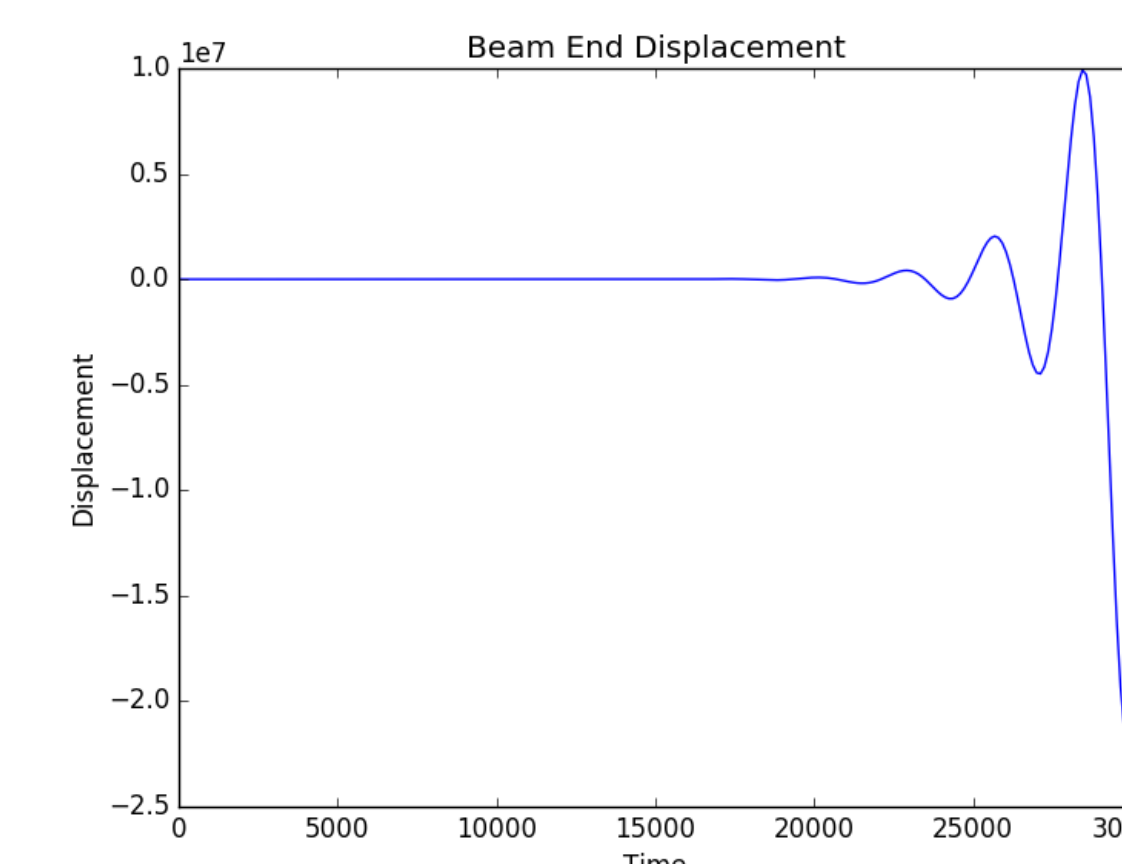
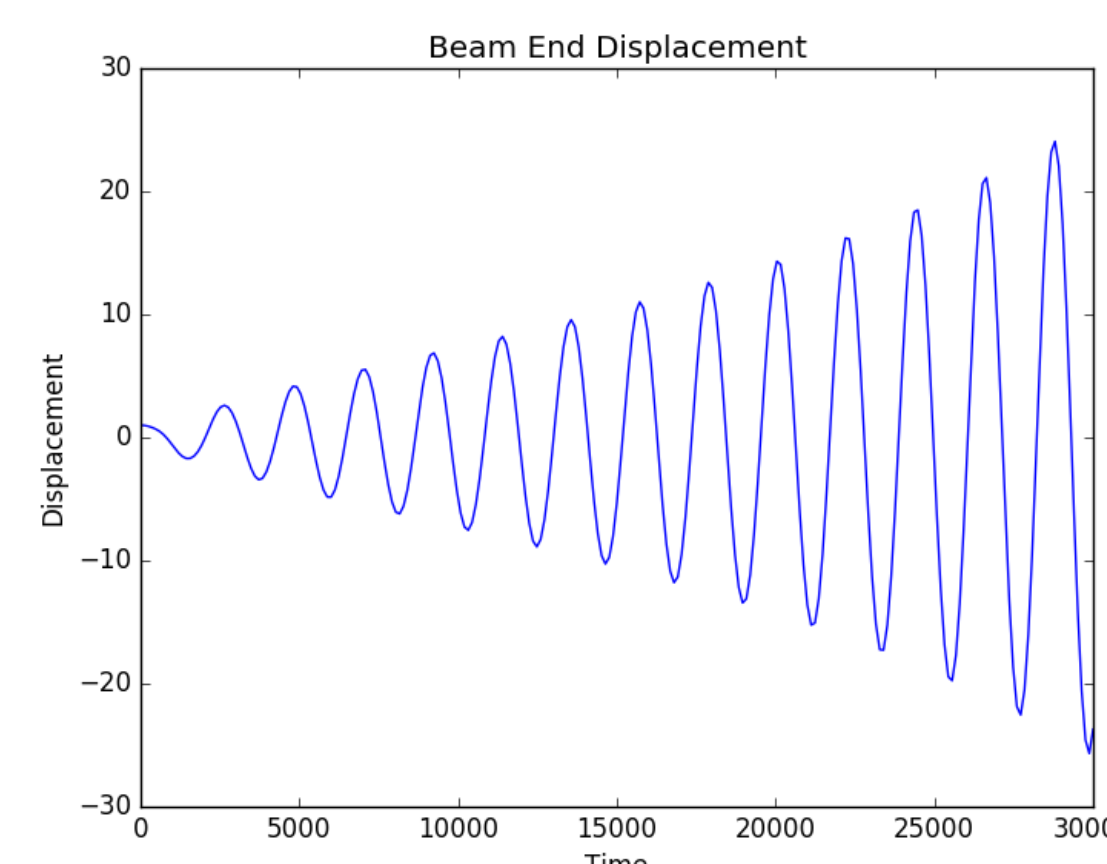
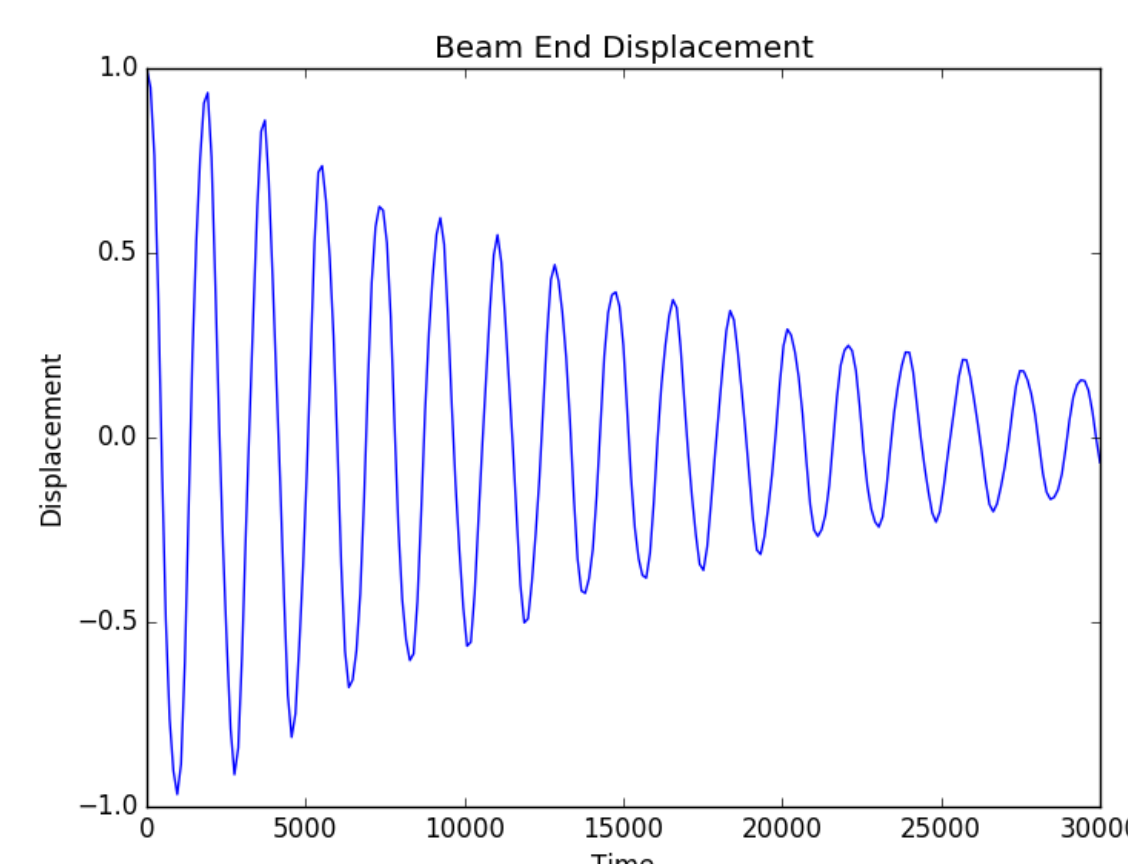
$$u(x, t) \approx e^{-i\omega t} \sum_{i=1}^N S_n(x)$$

- Presence of positive imaginary component of eigenvalue indicates instability.

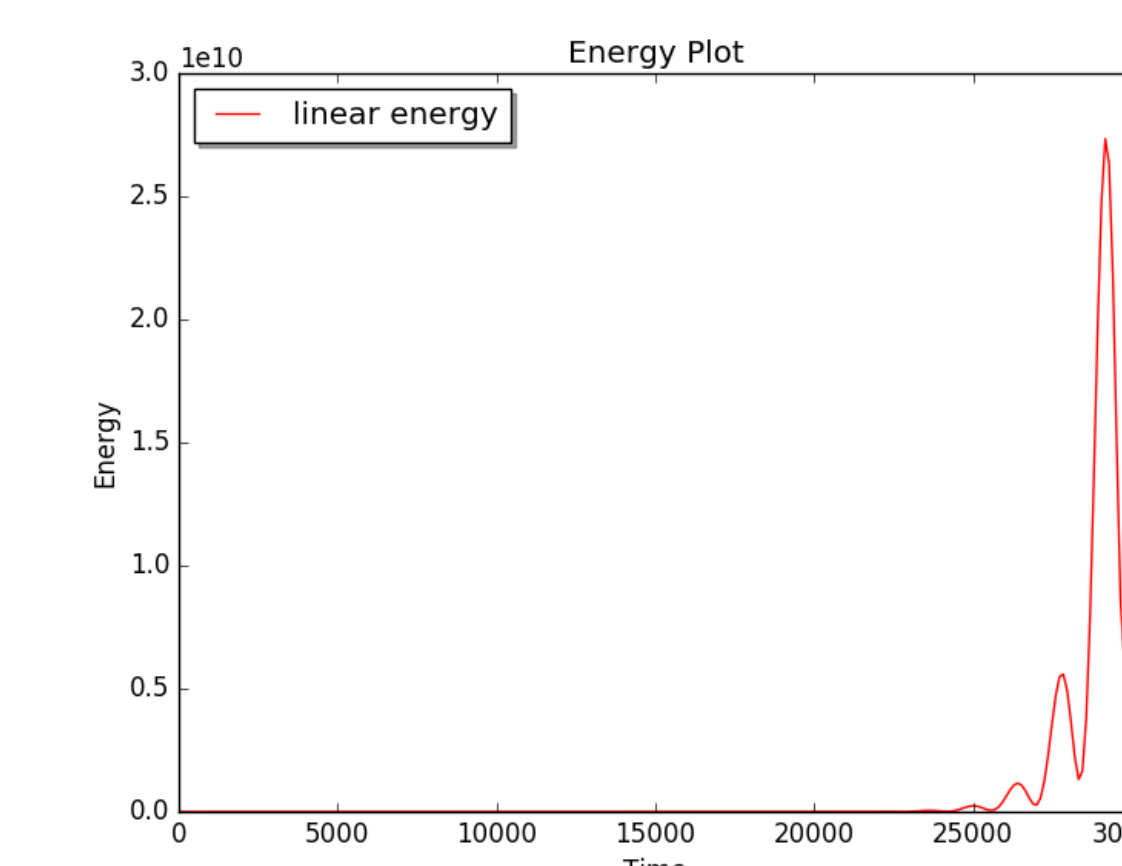
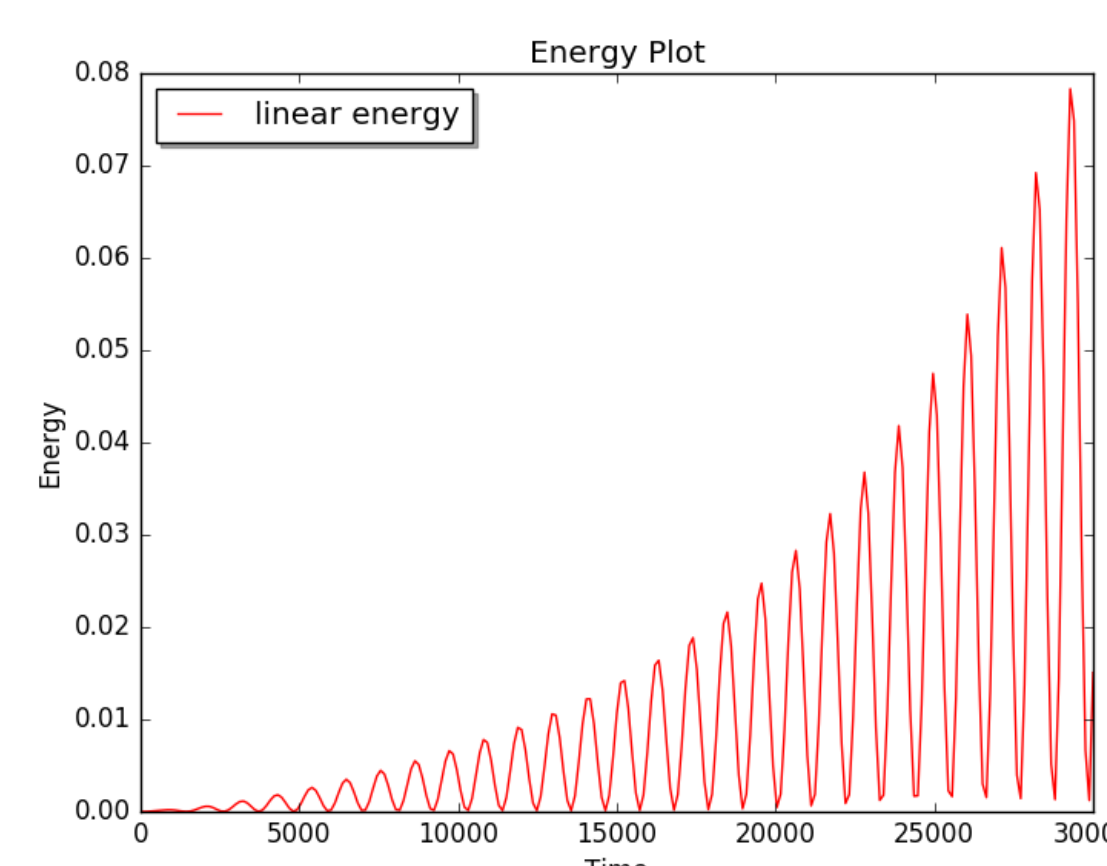
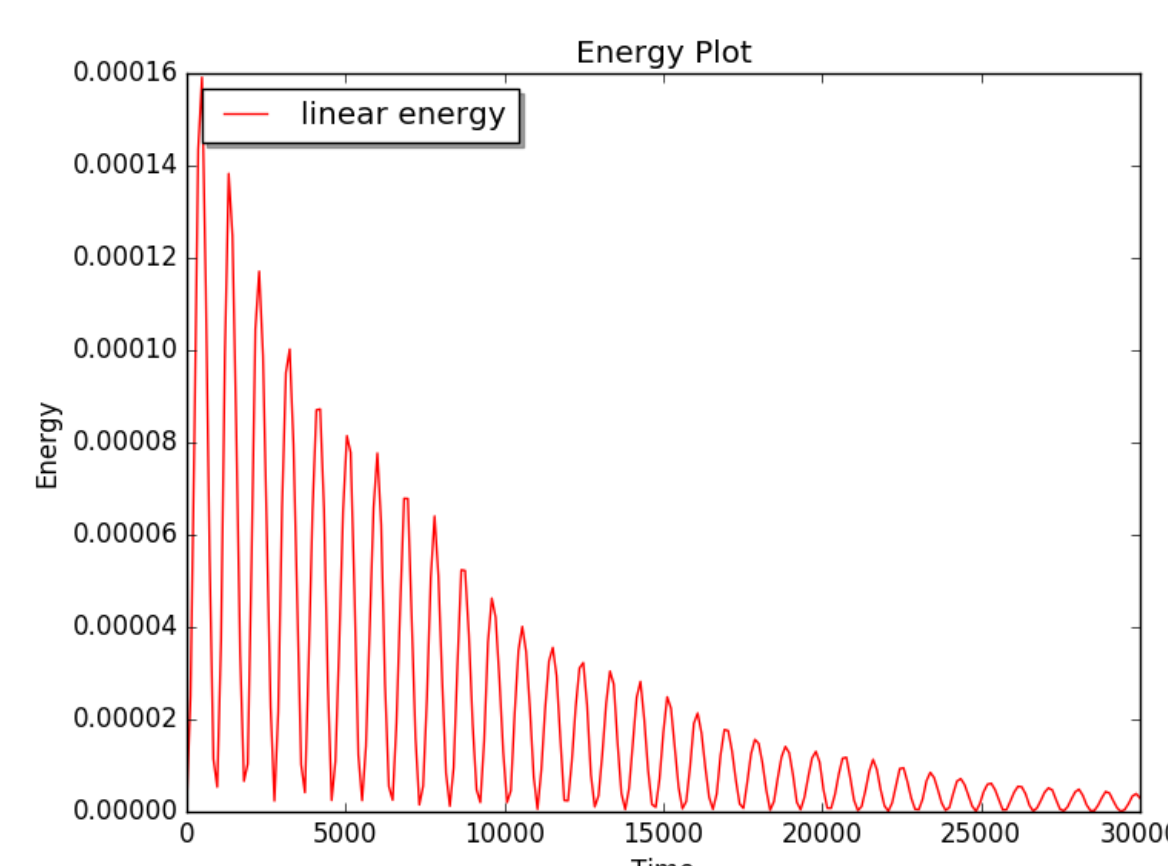
Parameter Studies

$$\text{Initial displacement: } u_0(x) = \frac{1}{3} \left(\frac{x}{L}\right)^4 - \frac{4}{3} \left(\frac{x}{L}\right)^3 + 2 \left(\frac{x}{L}\right)^2,$$

$$\text{Initial velocity: } u_1(x) = 0$$



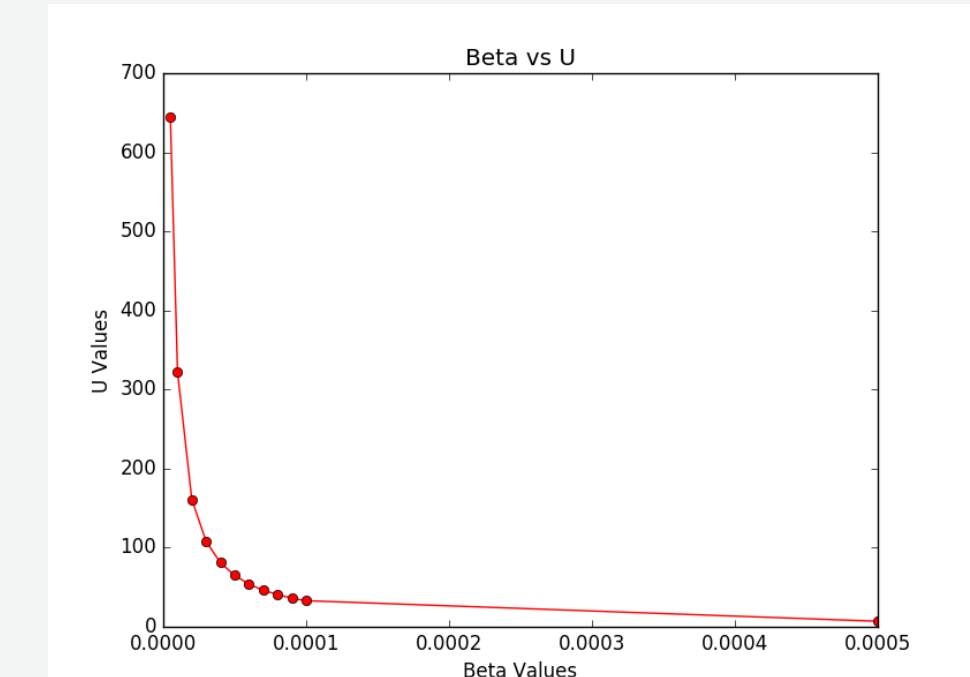
Tip displacements for $L = 100, 200,$ and 300 with $U = 3.25$



Energy plots for $L = 100, 200,$ and 300 with $U = 3.25$

Results

- Longer beam lengths flutter for smaller U .
- Region of stability for β versus U behavior: $\beta \propto \frac{1}{U}$



- Method of modal analysis corroborates with the finite difference method when predicting the onset of flutter.
- Piston theory yields predictions which agree with physical observations for $U > \sqrt{2}$.
- For a fluttering beam, the relationship between U_{crit} and $k_{0,crit}$ is linear

Future Work

- Finite Element Method
 - Problem is written in *variational* form
 - Spatial domain is divided into small "elements"
 - Polynomial basis functions on each element
 - Problem reduces to a system of linear equations
- Other piston theoretic RHS
 - Low supersonic Piston Theory

$$p(x, t) = p_0 - \beta_1(U) + \beta_2(U) \int_0^x \partial_t [u_t + U u_x] d\xi$$

- Nonlinear Piston Theory

$$p(x, t) = p_0 - \sum_{j=1}^3 \beta_j(U) (u_t + U u_x)^j$$

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Acknowledgements

The first and third authors acknowledge the financial support of the College of Charleston Office of Undergraduate Research and Creative Activities (SURF Grant No. SU2016-0029) and the second and fourth authors are supported in part by the National Science Foundation with grant NSF-DMS-1504697.

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