Computational Methods for Predicting Aeroelastic Flutter

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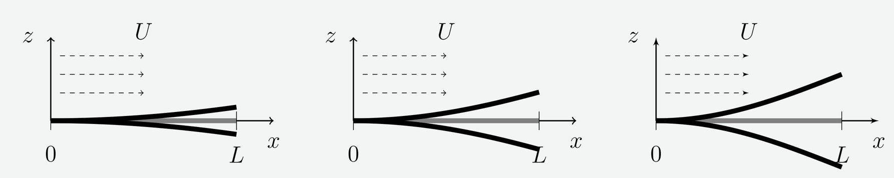
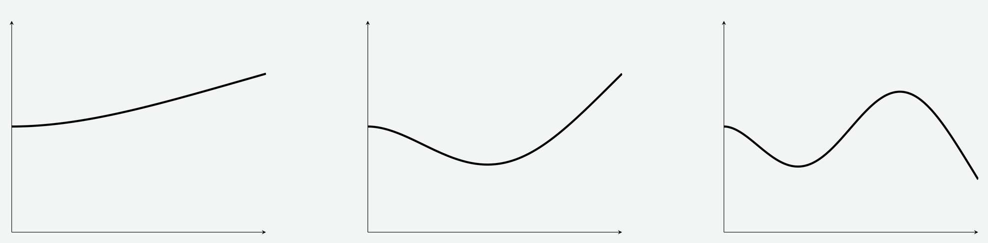
Introduction

- Flutter is a dynamic instability of an elastic structure in a fluid flow, caused by positive feedback between the body's deflection and the force exerted by the fluid flow.



- The elastic body has an infinite number of natural ways in which it wants to deflect due to external stresses.



Goals

- Understand parameter relationships
- Predict the onset of flutter using various computational methods
- Assess the validity of the piston-theoretic model.

Model

- Standard linear beam model with piston theoretic RHS.
- Piston Theory: assume that pressure on surface of plate can be treated as a piston acting on a narrow column of air above the plate.

$$\begin{cases} u_{tt} + D\partial_x^4 u + k_0 u_t = -\beta\big(u_t + U u_x\big) \\ u(t=0) = u_0; \quad u_t(t=0) = u_1 \\ BC(0); \quad BC(L). \end{cases} \qquad (1)$$

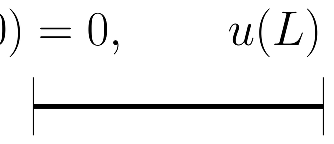
- Piston Theoretic RHS

$$\frac{-\mu U}{\sqrt[2]{U^2-1}}(u_t + U u_x)$$

- Parameter Log
  - D: Stiffness parameter
  - k_0: Damping
  - U: Fluid Flow Velocity
  - L: Length of Beam

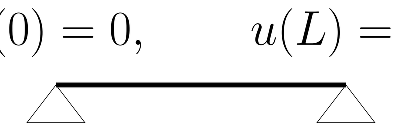
Clamped-Clamped BC:

$$u(0) = u_x(0) = 0, \qquad u(L) = u_x(L) = 0$$



Hinged-Hinged BC:

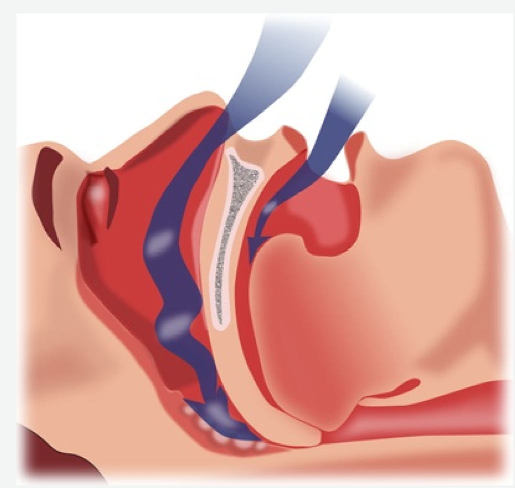
$$u(0) = u_{xx}(0) = 0, \qquad u(L) = u_{xx}(L) = 0$$



Clamped-Free BC:

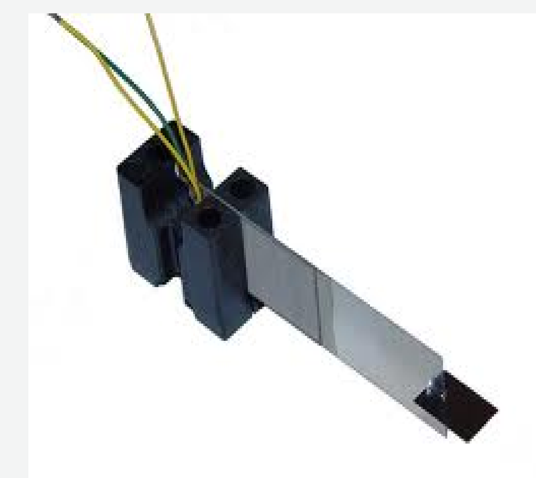
$$u(0) = u_x(0) = 0, \qquad u_{xx}(L) = u_{xxx}(L) = 0$$

Motivation and Applications



- Aerospace: flaps and flags
- Biology: snoring and sleep apnea
- Piezoelectric Energy Harvesting

Clamped-free configuration not well understood. Peculiarities include: low flutter velocities and large displacements in the unstable regime.



Spatial and Temporal Discretization

- Approximate spatial derivatives using second-order Taylor series expansions:

$$\partial_x u \approx \frac{u(x+h) - u(x-h)}{2h}, \quad \partial_x^2 u \approx \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}$$

$$\partial_x^4 u \approx \frac{u(x+2h) - 4u(x+h) + 6u(x) - 4u(x-h) + u(x-2h)}{h^4}$$

- Boundary conditions implemented using ghost values outside domain.
- Model is reduced to first-order system in time:

$$\begin{bmatrix} u \\ v \end{bmatrix}' = \begin{bmatrix} v \\ -D\partial_x^4 u - k_0 v - \beta\big(v + U u_x\big) \end{bmatrix}$$

- Time integration: makes use of `scipy.integrate.ode` python package, employing backward differentiation formulas (BDFs).

Modal Analysis

- Separation of variables for $u_{tt} + D u_{xxxx} = 0$ in vacuo: assume $u(x,t) = S(x)f(t)$

$$S_{xxxx} = k_m^4 S, \qquad f_{tt} = -\omega_m^2 f$$

- Use Euler identity $e^{ix} = \cos x + i \sin x$ to derive our basis functions $\cos$, $\sin$, $\cosh$, and $\sinh$.
- Linear combinations of trig and hyperbolic trig functions comprise the basis functions/mode shapes:

$$S_n(x) = \cos(k_n x) - \cosh(k_n x) - d_n\left[\sin(k_n x) - \sinh(k_n x)\right]$$

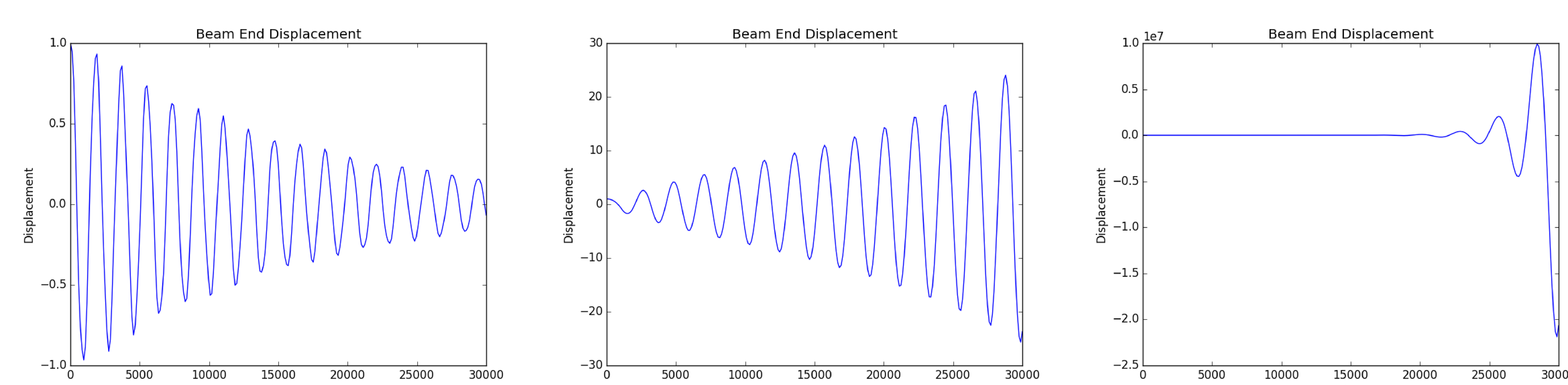
- Solve eigenvalue problem to find shifted freqeuncies (modes): assume

$$u(x,t) \approx e^{-i\bar{\omega}t}\sum_{i=1}^{N} S_n(x)$$

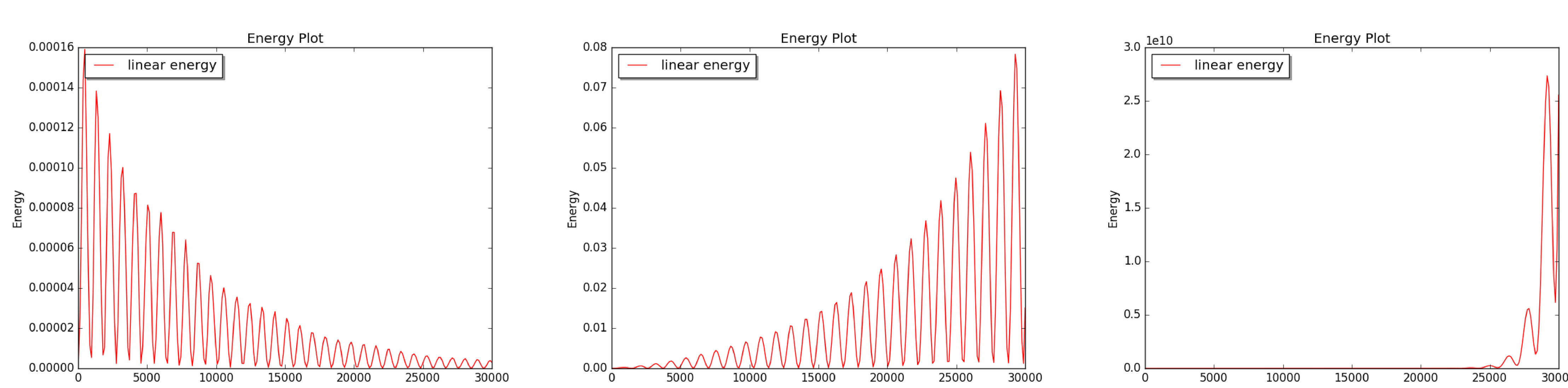
- Presence of positive imaginary component of eigenvalue indicates instability.

Parameter Studies

Initial displacement: $u_0(x) = \frac{1}{3}\left(\frac{x}{L}\right)^4 - \frac{4}{3}\left(\frac{x}{L}\right)^3 + 2\left(\frac{x}{L}\right)^2$, Initial velocity: $u_1(x) = 0$



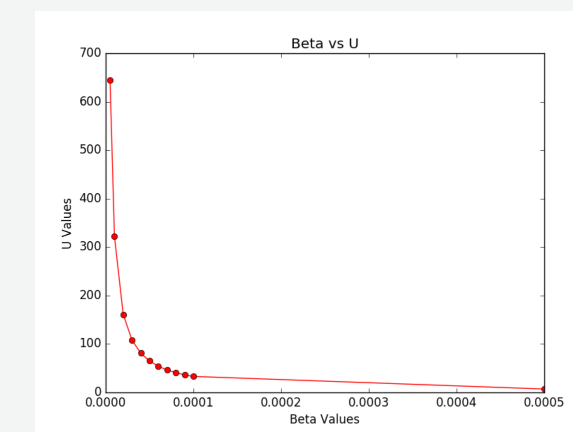
Tip displacements for $L = 100, 200$, and 300 with $U = 3.25$



Energy plots for $L = 100, 200$, and 300 with $U = 3.25$

Results

- Longer beam lengths flutter for smaller U.
- Region of stability for β versus U behavior: $\beta \propto \frac{1}{U}$



- Method of modal analysis corroborates with the finite difference method when predicting the onset of flutter.
- Piston theory yields predictions which agree with physical observations for $U > \sqrt{2}$.
- For a fluttering beam, the relationship between U_{crit} and $k_{0,crit}$ is linear

Future Work

- Finite Element Method
  - Problem is written in *variational* form
  - Spatial domain is divided into small "elements"
  - Polynomial basis functions on each element
  - Problem reduces to a system of linear equations
- Other piston theoretic RHS
  - Low supersonic Piston Theory

$$p(x,t) = p_0 - \beta_1(U) + \beta_2(U)\int_0^x \partial_t[u_t + U u_x]d\xi$$

  - Nonlinear Piston Theory

$$p(x,t) = p_o - \sum_{j=1}^{3}\beta_j(U)(u_t + U u_x)^j$$

References

[1] Chueshov, I., Dowell, E.H., Lasiecka, I. and Webster, J.T., 2016. Mathematical aeroelasticity: A survey. *Journal MESA*, 7(1), pp.5-29.

[2] E. Dowell, A Modern Course in Aeroelasticity, *Kluwer Academic Publishers*, 2004.

[3] Lasiecka, I. and Webster, J.T., 2016. Feedback stabilization of a fluttering panel in an inviscid subsonic potential flow. *SIAM Journal on Mathematical Analysis*, 48(3), pp.1848–1891.

[4] Lighthill, M. J., 1953. Oscillating airfoils at high mach number. *Journal of the Aeronautical Sciences*, 20(6), pp. 402–406.

[5] V. Vedeneev, Effect of damping on flutter of simply supported and clamped panels at low supersonic speeds ,*J. of Fluids and Structures*, 40 (2013), pp. 366–372.

[6] V.V. Vedeneev, Panel flutter at low supersonic speeds, *J. Fluids and Structures*, 29 (2012), pp. 79–96.

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